Mechanics M3 Mark scheme

Question	Scheme	Marks
1	(30° or θ for the first 3 lines)	
	$R\sin 30° = mg$	M1 A1
	$R\cos 30° = m\left(r\cos 30°\right)\omega^2$	M1 A1 A1
	$\omega^2 = \dfrac{R}{mr} = \dfrac{g}{r\sin 30}$	DM1
	$\omega = \sqrt{\dfrac{2g}{r}}$	A1
	Time $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{r}{2g}} = \pi\sqrt{\dfrac{2r}{g}}$ *	A1 cso
		(8)
	Alternative:	
	Resolve perpendicular to the reaction:	
	$mg\cos 30 = m \times rad \times \omega^2 \cos 60$	M2 A1 (LHS) A1 (RHS)
	$= mr\cos 30\, \omega^2 \cos 60$	A1
	Obtain ω	M1 A1
	Correct time	A1
		(8)
		(8 marks)

Notes:

M1: Resolving vertically   30° or θ

A1: Correct equation   30° or θ

M1: Attempting an equation of motion along the radius, acceleration in either form   30° or θ
Allow with r for radius.

A1: LHS correct   30° or θ

A1: RHS correct, 30° or θ but not r for radius.

DM1: Obtaining an expression for ω^2 or for v^2 **and** the length of the path   30° or θ Dependent on both previous M marks.

A1: Correct expression for ω Must have the numerical value for the trig function now.

A1cso: Deducing the GIVEN answer.

Question	Scheme	Marks
2(a)	$$F = \dfrac{K}{x^2}$$	
	$x = R \Rightarrow F = mg \qquad \therefore mg = \dfrac{K}{R^2}$	M1
	$K = mgR^2$ ✱	A1
		(2)
(b)	$\dfrac{mgR^2}{x^2} = -mv\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1
	$g\displaystyle\int \dfrac{R^2}{x^2}\,\mathrm{d}x = -\int v\,\mathrm{d}v$	
	$-g\dfrac{R^2}{x} = -\dfrac{1}{2}v^2 \ (+c)$	dM1 A1ft
	$x = 3R,\, v = V \Rightarrow -g\dfrac{R^2}{3R} = -\dfrac{1}{2}V^2 + c$	M1
	$c = -\dfrac{Rg}{3} + \dfrac{1}{2}V^2$	A1
	$x = R \Rightarrow \dfrac{1}{2}v^2 = -\dfrac{Rg}{3} + \dfrac{1}{2}V^2 + g\dfrac{R^2}{R}$	M1
	$v^2 = V^2 + \dfrac{4Rg}{3}$	
	$v = \sqrt{V^2 + \dfrac{4Rg}{3}}$	A1 cso
		(7)
		(9 marks)

Notes:

(a)

M1: Setting $F = mg$ **and** $x = R$

A1: Deducing the GIVEN answer

(b)

M1: Attempting an equation of motion with acceleration in the form $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$. The minus sign may be missing.

dM1: Attempting the integration.

A1ft: Correct integration, follow through on a missing minus sign from line 1, constant of integration may be missing.

M1: Substituting $x = 3R, v = V$ to obtain an equation for c

A1: Correct expression for c.

M1: Substituting $x = R$ and their expression for c.

A1: Correct expression for v, any equivalent form.

Question	Scheme	Marks
3(a)	$\dfrac{dv}{dt} = -2(t+4)^{-\frac{1}{2}}$	M1
	$v = -\int 2(t+4)^{-\frac{1}{2}}\, dt$	
	$v = -4(t+4)^{\frac{1}{2}}\ (+c)$	dM1 A1
	$t = 0,\, v = 8 \Rightarrow c = 16$	M1
	$v = 16 - 4(t+4)^{\frac{1}{2}}\ (m\,s^{-1})\ *$	A1 **cso**
		(5)
(b)	$v = 0 \quad 16 = 4(t+4)^{\frac{1}{2}}$	M1
	$16 = t + 4 \quad t = 12$	A1
	$x = 4\int \left(4 - (t+4)^{\frac{1}{2}} \right) dt$	
	$x = 4\left(4t - \dfrac{2}{3}(t+4)^{\frac{3}{2}} \right)\ (+d)$	M1 A1
	$t = 0,\ x = 0 \quad d = 4 \times \dfrac{2}{3} \times 4^{\frac{3}{2}} = \dfrac{64}{3} \qquad$ oe	A1
	$t = 12 \quad x = 4\left(4 \times 12 - \dfrac{2}{3} \times 16^{\frac{3}{2}} \right) + \dfrac{64}{3} = 42\dfrac{2}{3}\ (m) \qquad$ oe eg 43 or better	dM1 A1
		(7)
		(12 marks)

Notes:

(a)

M1: Attempting an expression for the acceleration in the form $\dfrac{dv}{dt}$; minus may be omitted.

DM1: Attempting the integration

A1: Correct integration, constant of integration may be omitted (no ft)

M1: Using the initial conditions to obtain a value for the constant of integration

A1: **cso.** Substitute the value of c and obtain the final GIVEN answer

(b)

M1: Setting the **given** expression for v equal to 0

A1: Solving to get $t = 12$

M1: Setting $v = \dfrac{dx}{dt}$ and attempting the integration wrt t. At least one term must clearly be integrated.

A1: Correct integration, constant may be omitted.

Question 3 notes *continued*
M1: Substituting $t = 0$, $x = 0$ and obtaining the correct value of d. Any equivalent number, inc decimals.
dM1: Substituting their value for t and obtaining a value for the required distance. Dependent on the second M mark.
A1: Correct final answer, any equivalent form.

Question	Scheme	Marks
4(a)	Energy to top: $\frac{1}{2} \times 3m \times u^2 - \frac{1}{2} \times 3mv^2 = 3mga$	M1 A1
	NL2 at top: $T + 3mg = 3m\frac{v^2}{a}$	M1 A1
	$T = 3m\frac{u^2}{a} - 6mg - 3mg$	dM1
	$T \geqslant 0 \implies \frac{u^2}{a} \geqslant 3g$	M1
	$u^2 \geqslant 3ag$	A1 cso
		(7)
(b)	Tension at bottom:	
	$\frac{1}{2} \times 3m \times V^2 - \frac{1}{2} \times 3mu^2 = 3mga$	M1
	$T_{max} - 3mg = 3m\frac{V^2}{a}$	M1
	$T_{max} = 3mg + 6mg + 3m\frac{u^2}{a}$	A1
	$T_{min} = 3m\frac{u^2}{a} - 9mg$	
	$9mg + 3m\frac{u^2}{a} = 3\left(3m\frac{u^2}{a} - 9mg\right)$	dM1
	$u^2 = 6ag$ **＊**	A1 cso
		(5)
		(12 marks)

Notes:

(a)

M1: Attempting an energy equation, can be to a general point for this mark. Mass can be missing but use of $v^2 = u^2 + 2as$ scores M0

A1: Correct equation from *A* to the top.

M1: Attempting an equation of motion along the radius at the top, acceleration in either form.

A1: Correct equation, acceleration in form $\dfrac{v^2}{r}$

dM1: Eliminate v^2 to obtain an expression for *T* dependent on both previous M marks.

M1: Use $T \geqslant 0$ at top to obtain an inequality connecting *a, g* and *u*

A1: Re-arrange to obtain the GIVEN answer.

Question 4 notes *continued*

(b)

M1: Attempting an energy equation to the bottom, maybe from *A* or from the top.

M1: Attempting an equation of motion along the radius at the bottom.

A1: Correct expression for the max tension.

dM1: Forming an equation connecting *their* tension at the top with *their* tension at the bottom. If the 3 is multiplying the wrong tension this mark can still be gained. Dependent on both previous M marks.

A1: **cso.** Obtaining the GIVEN answer.

Question	Scheme	Marks
5(a)	$T = \dfrac{20e}{2} = \dfrac{15(1.8-e)}{1.2}$	M1A1
	$10e \times 1.2 = 15(1.8-e)$	
	$e = 1$	A1
	$AO = 3\,\text{m}$ **✳**	A1cso
		(4)
(b)	$0.5\ddot{x} = \dfrac{20(1-x)}{2} - \dfrac{15(0.8+x)}{1.2}$	M1 A1 A1
	$\ddot{x} = -45x \quad \therefore \text{SHM}$	A1 cso
		(4)
(c)	String becomes slack when $x = (-)0.8$ (allow wo sign due to symmetry)	B1
	$v^2 = \omega^2\left(a^2 - x^2\right)$	
	$v^2 = 45\left(1 - 0.8^2\right) \quad (=16.2)$	M1 A1 ft
	$v = 4.024... \;\; \text{m s}^{-1}$ (4.0 or better)	A1ft
		(4)
(d)	$\dfrac{1}{2} \times \dfrac{20y^2}{2} - \dfrac{1}{2} \times \dfrac{20 \times 1.8^2}{2} = \dfrac{1}{2} \times 0.5 \times 16.2$ ft on v	M1 A1 A1 ft
	$20y^2 - 64.8 = 16.2$	
	$y^2 = 4.05 \quad y = 2.012....$	A1
	Distance $DB = \left|5 - 4.012...\right| = 0.988...\,\text{m}$ (accept 0.99 or better)	A1ft
	Alternative	
	$0.5\text{a} = -10(1.8 + x)$	
	$v\dfrac{\text{d}v}{\text{d}x} = -36 - 10x$	
	$\int v\,\text{d}v = -\int (36+10x)\,\text{d}x$	
	$\dfrac{v^2}{2} = -36x + 5x^2 + c$	M1 A1
	$x = 0, \; v = \dfrac{9\sqrt{5}}{5} \; \therefore \; c = 8.1$	A1
	Then $v = 0$ etc	M1 A1
		(5)
		(17 marks)

Question 5 *continued*

Notes:

(a)

M1: Attempting to obtain and equate the tensions in the two parts of the string.

A1: Correct equation, extension in *AP* or *BP* can be used or use *OA* as the unknown.

A1: Obtaining the correct extension in either string (ext in *BP* =0.8 m) or another useful distance.

A1: **cso.** Obtaining the correct GIVEN answer.

(b)

M1: Forming an equation of motion at a general point. There must be a difference of   tensions, both with the variable. May have *m* instead of 0.5 Accel can be *a*.

A1 A1: Deduct 1 for each error, *m* or 0.5 allowed, acceleration to be $\ddot{x}$ now.

A1: **cso** Correct equation in the required form, with a concluding statement; *m* or 0.5   allowed.

Question 5 notes *continued*

(c)

B1: For $x = \pm 0.8$ Need not be shown explicitly.

M1: Using $v^2 = \omega^2\left(a^2 - x^2\right)$ with *their* (numerical) ω and their x

A1ft: Equation with correct numbers ft their ω

A1ft: Correct value for v 2sf or better or exact.

(d)

M1: Attempting an energy equation with 2 EPE terms and a KE term.

A1: 2 correct terms   may have $(1.8 + x)$ instead of *y*.

A1ft: Completely correct equation, follow through their *v* from (c)

A1: Correct value for distance travelled after *PB* became slack. $x = 0.21$

A1ft: Complete to the distance *DB*. Follow through their distance travelled after *PB* became slack.

Question	Scheme	Marks
6(a)	$\text{Vol} = \pi\int_0^2 \left(x^2+3\right)^2 \, \mathrm{d}x$	M1
	$= \pi\int_0^2 \left(x^4+6x^2+9\right)\mathrm{d}x$	
	$= \pi\left[\frac{1}{5}x^5+2x^3+9x\right]_0^2$	dM1 A1
	$= \frac{202}{5}\pi \text{ cm}^3$ $*$	A1
		(4)
(b)	$\pi\int_0^2 x\left(x^2+3\right)^2 \mathrm{d}x = \pi\int_0^2\left(x^5+6x^3+9x\right)\mathrm{d}x$	M1
	$= \pi\left[\frac{1}{6}x^6+\frac{3}{2}x^4+\frac{9}{2}x^2\right]_0^2$	A1
	$= \frac{158}{3}\pi$ (Or by chain rule or substitution)	A1
	$\text{C of m} = \frac{158}{3}\times\frac{5}{202}, \ =1.3036...=1.30$ cm	M1 A1
		(5)
(c)	Mass ratio $\quad 2\times\frac{202}{5}\pi \qquad \frac{1}{3}\pi\times7^2\times6 \qquad \left(\frac{404}{5}+98\right)\pi$	B1
	Dist from V $\qquad$ 6.7 $\qquad\qquad$ 4.5 $\qquad\qquad\qquad$ $\overline{x}$	B1
	$\frac{404}{5}\times6.7+98\times4.5 = \left(\frac{404}{5}+98\right)\overline{x}$	M1 A1 ft
	$\overline{x} = \dfrac{\frac{404}{5}\times6.7+98\times4.5}{\left(\frac{404}{5}+98\right)} = 5.494....=5.5$ cm $\quad$ Accept 5.49 or better	A1
		(5)
(d)	$\tan\theta = \dfrac{6-\overline{x}}{7} = \dfrac{0.5058...}{7}$	M1
	$\alpha = \tan^{-1}\left(\dfrac{6}{7}\right) - \tan^{-1}\left(\dfrac{0.5058...}{7}\right) = 36.468...° = 36°$ $\quad$ or better	M1 A1
		(3)
		(17 marks)
Notes:		
(a)		
M1:	Using $\pi\int y^2\mathrm{d}x$ with the equation of the curve, no limits needed	

Question 6 notes *continued*
dM1: Integrating their expression for the volume.   **A1:** Correct integration inc limits now.   **A1:** Substituting the limits to obtain the GIVEN answer.
(b)   **M1:** Using $(\pi)\int xy^2 dx$ with the equation of the curve, no limits needed, π can be omitted.   **A1:** Correct integration, including limits; no substitution needed for this mark.   **A1:** Correct substitution of limits.   **M1:** Use of $\dfrac{\pi\int xy^2 dx}{\pi\int y^2 dx}$ with their $\pi\int xy^2 dx$. π must be seen in both numerator and denominator or in neither.   **A1:** **cso.** Correct answer. Must be 1.30
(c)   **B1:** Correct mass ratio.   **B1:** Correct distances, from V or any other point, provided consistent.   **M1:** Attempting a moments equation.   **A1ft:** Correct equation, follow through their distances and mass ratio.   **A1:** Correct distance from V
(d)   **M1:** Attempting the tan of an appropriate angle, numbers either way up.   **M1:** Attempting to obtain the required angle.   **A1:** Correct final answer 2sf or more.